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$$(\zeta_1^3) = \pm \frac{.6745\,\varepsilon_1^3}{m} \sqrt{(15[p])}. \tag{94}$$

This is the limit within which there is about an even chance for ζ_1^3 to fall if the inequality is fortuitous. In our present example it gives

$$(\zeta_1^3) = \pm \frac{.6745}{14} \sqrt{[15 \times 46(89.77)^3]} = 1076.4.$$

The actual value by (89) is $\zeta_1^3 = -1464.4$. The excess is not very great, and taken in connection with what we know of the nature of theodolite observations, it does not justify the belief that there was any real want of symmetry in the law of error.

Granting that the true law is symmetrical, the most probable value of the observed quantity is the ultimate arith. mean, that is, the mean of all the possible values, each taken a number of times proportional to the probability of its occurrence. But this ultimate mean cannot be found without taking a number of observations which is very large indeed, or infinite. We have to infer its probable value from a moderate number, in the foregoing case only $m = 14$. Owing to the small number of observations, the distribution of the residual errors is irregular and unsymmetrical, and the law of error of the mean, so far as it can be deduced from the observations, is an unsymmetrical law. Hence, the most probable value of the ultimate arith. mean is not the apparent mean u_0, but u_0 plus the correction Δ_0 due to the c. m. inequality ζ_0. The resulting value of the mean and its probable errors as exemplified in (93) constitute, as it seems to me, the best determination we can make of the most probable value of the observed quantity.

CORRESPONDENCE.

Editor Analyst:

I desire to make an additional remark in regard to the paradox in the Query by Professor Johnson. (See ANALYST, No. 2, p. 44.)

The principle which seems to be violated in the Ex. given is this:—If $u = f(a, x)$,.. (1) and $du \div dx = f'(a, x)$,.. (2) and we assign to a such value as will reduce u to a constant or to zero, then the same value ought to reduce $f'(a, x)$ to zero. If it does not, the paradox must arise from a misinterpretation of the result, or of some of the *singular forms* that enter into the work.

Now although $\sin \infty$ and $\cos \infty$ may each be regarded as zero, in the sense that any constant may be regarded as zero in comparison with another quantity which is infinitely greater, we cannot admit 0 as an absolute value in violation of the principle $\sin^2\infty + \cos^2\infty = 1$.

The paradox clearly arises from a misinterpretation of the form $a \div \infty$.

If $u = a^{-1}\sin ax$, then $du \div dx = \cos ax$ is true for all definite values of a, *however great;* and there is no difficulty in the interpretation of the result. But when $a = \infty$ both u and $du \div dx$ become indefinite.

If however $a = \infty$ renders u *infinitesimal,* it still remains a function of x, and there seems to be no good reason why we should then expect $du \div dx$ to equal zero, *independently* of x. Let $u = a^{-1}\tan ax$, then $du \div dx = \sec^2 ax$ which is always greater than unity, though u may be infinitesimal.

If we write $u = a \sin (x \div a) \ldots (1)$, and $du \div dx = \cos (x \div a)$ and then make $a = 0$, the paradox appears in another form. If $a = 0$ (absolutely) then $u = 0$ independently of x; and if $x \div 0$ is a *quantity* $= \infty$ the paradox remains. But if, as I have attempted to show (see ANALYST, for July 1881), $x \div 0$ is only *a symbol* of *impossible conditions,* and not one of quantity, the paradox disappears.

Reject the forms $a.0^{-1} = \infty$, and $a.\infty^{-1} = 0$, and substitute $a.\bigcirc^{-1} = \infty$, and $a.\infty^{-1} = \bigcirc$, $a.0^{-1}$ = sign of impossible conditions, and a host of paradoxes will vanish.

Note. ∞ and $\bigcirc$ are employed as symbols of variables, one increasing without limit and the other decreasing without limit.

C. H. JUDSON.

Greenville, S. C., March 10, 1883.

CORRECTION OF AN ERROR IN BARTLETT'S MECHANICS.—A remarkable error occurs in Bartlett's Analytical Mechanics, ninth edition, p. 178. The solution of the differential equation

$$\frac{d^2v_y}{dt^2} - m^2v_y = 0$$

is given as

$$v_y = a e^{mt+c},$$

(in which I have written m^2 for the constant $n^2(A-C)(C-B) \div (A.B)$, supposed positive) and a and c for a_y and c_y the constants of integration. The apparent presence of two constants appears to have induced the belief that the integral thus written was complete; whereas it can of course be written

$$v_y = a_y e^c e^{mt},$$

in which $a_y e^c$ is equivalent to a single constant. Thus the value given is only a particular integral, the complete integral being of the form

$$v_y = Ae^{mt} + Be^{-mt}.$$

The differential equation arises in the solution of simultaneous equations of the first order for the determination of v_x and v_y, and it may be remarked that both this and the preceding solution are faulty in presenting different sets of constants in the values of v_y and v_x, or in all four constants instead of two.

WM. WOOLSEY JOHNSON.